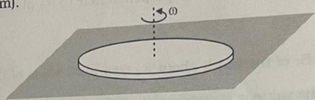
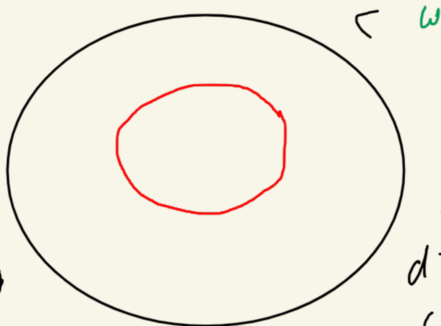
(A) $\frac{6g}{13}$ (B) $\frac{\ }{4}$

60. A uniform disc of radius R is set spinning with an angular velocity ω and then carefully placed on a horizontal surface with its base in contact with the surface as shown. How long will the disc be rotating on the surface if the coefficient of friction is equal to μ. (The pressure exerted by the disc on the surface can be regarded as uniform).



(A) $\frac{4\omega R}{3\mu g}$ (B) $\frac{3\omega R}{4\mu g}$ ✓ (C) $\frac{\omega R}{3\mu g}$ (D) $\frac{4\omega R}{\mu g}$



$\sigma = \frac{M}{\pi R^2}$

$T = 2\pi \mu g \times \frac{R^3}{3} \times \sigma$

$= \sigma \pi R^2 \times \frac{2}{3} \mu g$

$= \frac{2}{3} \mu M g R$

$T = I\alpha \Rightarrow \frac{2}{3} \mu M g R = \frac{MR^2}{2} \times \alpha$

$\Rightarrow \alpha = \frac{4}{3} \frac{\mu g}{R}$

$t = \frac{\omega_i}{\alpha} = \frac{3R\omega}{4\mu g}$

$dm = \sigma \times 2\pi r\, dr$

$dN = dm \times g$

$df = \mu\, dm \times g$

$dT = df \times r$

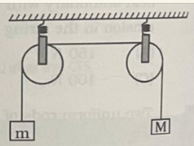
$= \int dT = \int_0^R \mu\, 2\pi r\, dr \times g \times r \times \sigma$

$\omega_f = \omega_i - \alpha t$, $\omega_f = 0$

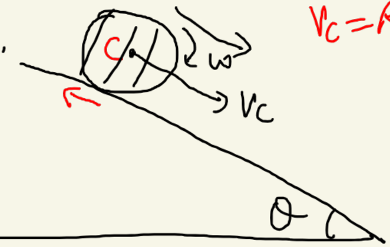
42. Each pulley shown in the given figure has radius r and moment of inertia I. The acceleration of blocks is:

(A) $\dfrac{(M-m)g}{\left(M+m+\dfrac{2I}{r^2}\right)}$ (B) $\dfrac{(M-m)g}{\left(M+m-\dfrac{2I}{r^2}\right)}$

(C) $\dfrac{(M-m)g}{\left(M+m+\dfrac{I}{r^2}\right)}$ (D) $\dfrac{(M-m)\,g}{\left(M+m-\dfrac{I}{r^2}\right)}$



Rolling down an inclined plane



$V_c = R\omega$

$a_c =$

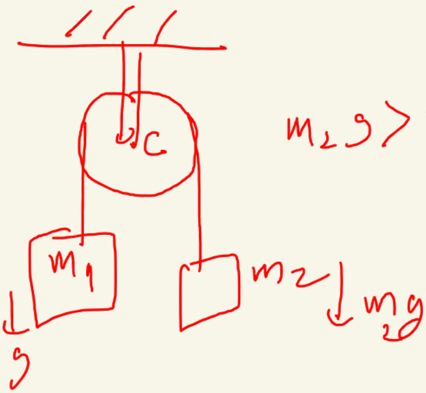
① $mg\sin\theta - f = m \times a_c$

② $f \times R = I \times \dfrac{a_c}{R}$ $\Rightarrow f = \dfrac{I}{R^2} a$

$mg\sin\theta = m a_c + \dfrac{I}{R^2} a_c$ → inertia

$a_c = \dfrac{mg\sin\theta}{m + I_c/R^2}$

Single pulley (massless)



$m_2 g > m_1 g$

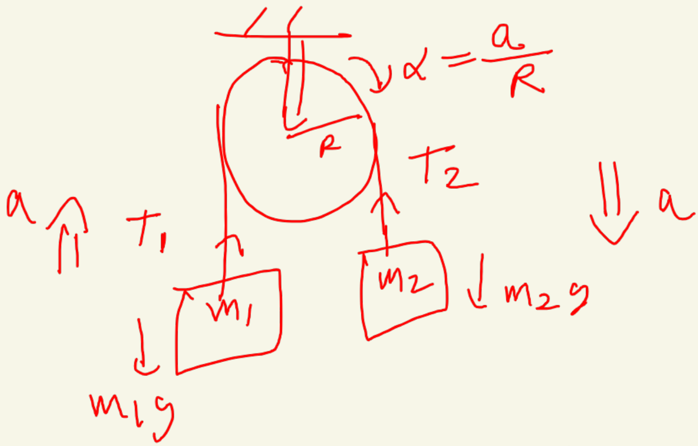
(i) Net force $= m_2 g - m_1 g$

$a = \dfrac{m_2 g - m_1 g}{(m_1 + m_2)}$

(ii) Pulley has a mass M_p, radius R.

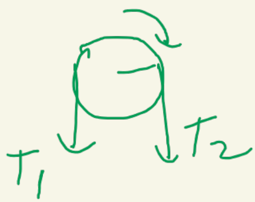
Mass equiv $= \dfrac{I_c}{R^2} = \dfrac{M' R^2}{2 \times R^2} = \dfrac{M'}{2}$

$a = \dfrac{(m_2 - m_1) g}{m_1 + m_2 + M'/2}$



FBDS m_2: $m_2 g - T_2 = m_2 a$

m_1: $T_1 - m_1 g = m_1 a$



Pulley: $(T_2 - T_1) R = I \times \frac{a}{R}$

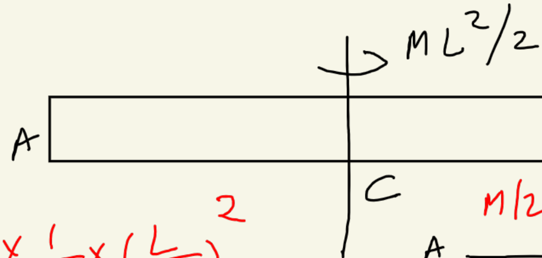
$\Rightarrow \quad T_2 - T_1 = \frac{I}{R^2} \times a$

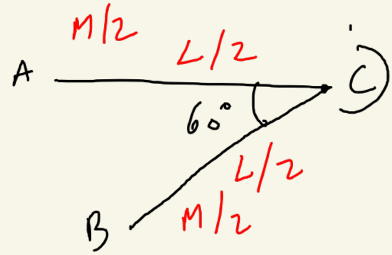
$$m_2(g-a) - m_1(g+a) = \frac{I}{R^2} a$$

$\Rightarrow \quad m_2 g - m_1 g = \left(m_1 + m_2 + \frac{I}{R^2}\right) a$

$\Rightarrow \quad a = \dfrac{(m_2 - m_1) g}{m_1 + m_2 + I/R^2}$

10. The moment of inertia of a rod about an axis through its centre and perpendicular to it is $\frac{1}{12}ML^2$ (where M is the mass and L is the length of the rod). The rod is bent in the middle so that the two halves make an angle of 60° with each other. The moment of inertia about the same axis would be $n\frac{ML^2}{12}$. Find n.



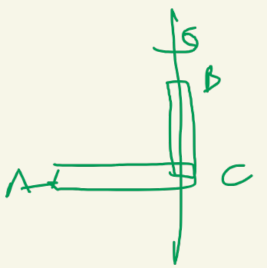


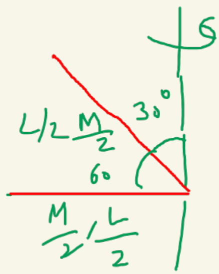
$$\cancel{2} \times \frac{M}{\cancel{2}} \times \frac{1}{3} \times \left(\frac{L}{2}\right)^2$$

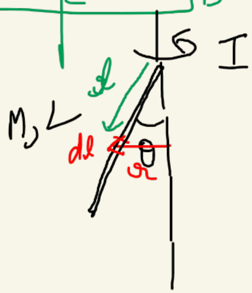
$$= M \times \frac{1}{3} \times \frac{L^2}{4} = \frac{ML^2}{12}$$

$n = 1$

Bent in the plane of







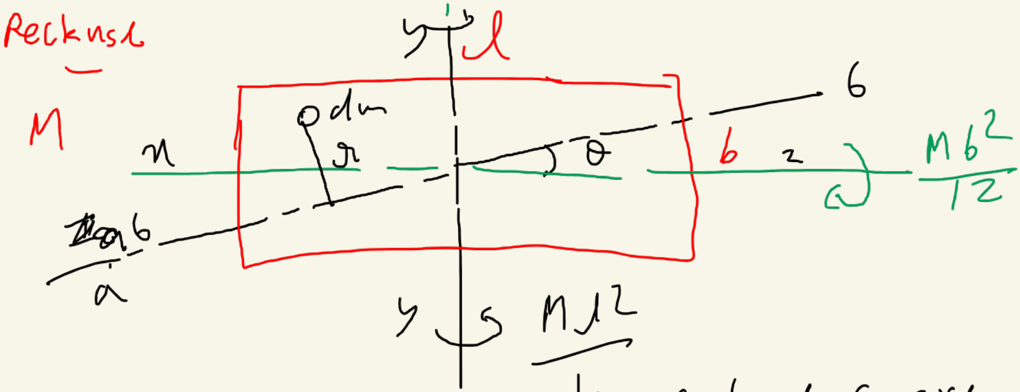
$$dm = \lambda\, dl, \quad \lambda = \frac{M}{L}$$

$$dI = dm \times (l \sin\theta)^2$$

$$dI = \frac{M}{L}\, dl \times l^2 \sin^2\theta$$

$$I = \int dI = \frac{M}{L}\sin^2\theta \times \int_0^L l^2\, dl$$

$$= \frac{ML^2}{3}\sin^2\theta$$

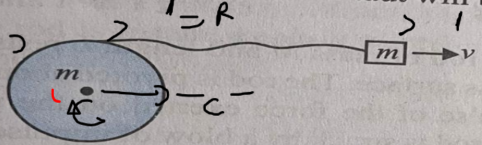


a-b is in the plane of the rectangle, so are

I_{xx}, I_{yy}

$$I_{ab} = I_{xx} \cos^2\theta + I_{yy} \sin^2\theta$$

(40) A block of mass m is attached to a pulley disc of equal mass m and radius r by means of a slack string as shown. The pulley is hinged about its centre on a horizontal table and the block is projected with an initial velocity of 5 m/s. Its velocity when the string becomes taut will be



(1) 3 m/s (2) 2.5 m/s

(3) $\frac{5}{3}$ m/s (4) $\frac{10}{3}$ m/s

1. Constraint equation 2) NLM 3) FBD

Constraint: $V_C = 0$, $v' = R\omega$

NLM: Ext force acting at C : ∴ $\vec{P}$ not conserved

Will $\vec{L}$ be conserved? L_C = Conserved

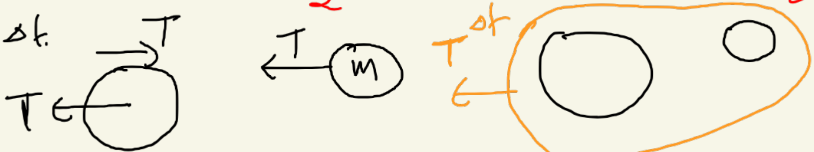
$\Rightarrow$ $mvR = mv'R + I\omega$

$\cancel{m} v \cancel{R} = \cancel{m} v' \cancel{R} + \frac{\cancel{m} R^{\cancel{2}}}{2} \times \frac{v'}{\cancel{R}}$

$\Rightarrow$ $v = v' + \frac{v'}{2}$ $\Rightarrow$ $v' = \frac{2}{3}v = \frac{10}{3}$ m/s

Suppose time is Δt.

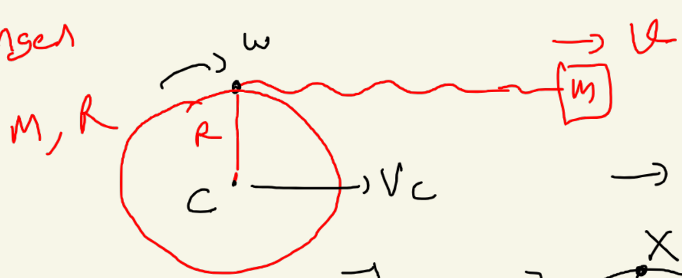
2nd Law

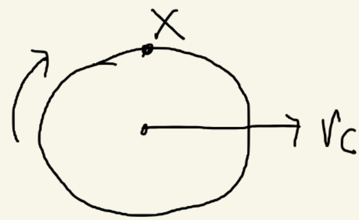


$T\Delta t = \underline{F_n \Delta t} = m(v - v') =$

$\Delta t = 10^{-5}$ s

Not hinged





Initial: $V_c = 0$, $\omega = 0$, $\vec{v}$

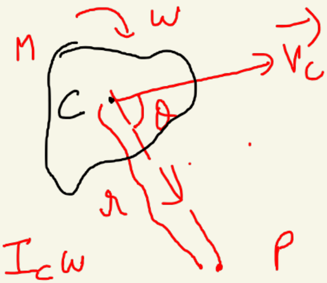
Final: v', ω, V_c

(1) $mv = mv' + MV_c$

(2) $v' = V_c + R\omega$

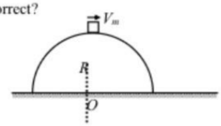
(3) $I\omega = MV_c \times R$

$$mv \times R = mv'R + I\omega$$



$$\vec{L}_P = I_c\omega + M \times V_c \sin\theta \times r$$

6. A block is kept on the top of a fixed smooth hemisphere with centre at O and radius R. Which of the following statement(s) is (are) correct?

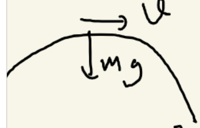


(A) If block is released from top position and it starts sliding down, then block will leave the surface at an angle $\cos^{-1}(2/3)$ with vertical

(B) In case of situation given in (A), block will leave the surface at an angle $\cos^{-1}\left(\frac{3}{4}\right)$ from vertical

(C) The minimum horizontal speed that if given to block at top so that it leaves the surface at top itself is equal to $\sqrt{Rg}$ ✓

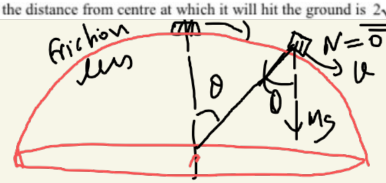
(D) If horizontal velocity twice of the minimum velocity required for block to leave the hemisphere at top point itself is given, then the distance from centre at which it will hit the ground is $2\sqrt{2}\,R$ ✓



$mg = \frac{mv^2}{R}$

$v = \sqrt{gR}$

A, C, D



(1) $mg\cos\theta = \frac{mv^2}{R}$

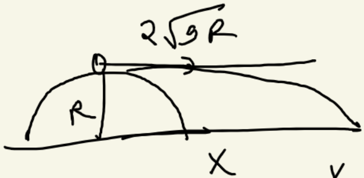
Conserve energy.

$mgR(1-\cos\theta) = \frac{1}{2}mv^2$

$\Rightarrow 2mg(1-\cos\theta) = \frac{mv^2}{R} = mg\cos\theta$

$\Rightarrow 2mg = 3mg\cos\theta \Rightarrow \cos\theta = \frac{2}{3} \quad \theta = \cos^{-1}\frac{2}{3}$

D.

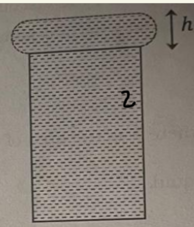


$R = \frac{1}{2}gt^2 \quad t = \sqrt{\frac{2R}{g}}$

$X = 2\sqrt{Rg} \times \sqrt{\frac{2R}{g}} = 2\sqrt{R\cancel{g} \times \frac{2R}{\cancel{g}}}$

$X = 2\sqrt{2}R$

15. When water is filled carefully in a glass, one can fill it to a height h above the rim of the glass due to the surface tension of water. To calculate h just before water starts flowing, model the shape of the water above the rim as a disc of thickness h having semicircular edges, as shown schematically in the figure. When the pressure of water at the bottom of this disc exceeds what can be withstood due to the surface tension, the water surface breaks near the rim and water starts flowing from there. If the density of water, its surface tension and the acceleration due to gravity are $10^3\ \text{kg m}^{-3}$, $0.07\ \text{Nm}^{-1}$ and $10\ \text{ms}^{-2}$, respectively, the value of h (in mm) is ________ . **[2020]**



$h = \sqrt{\dfrac{2T}{dg}}$ $\quad h\,dg = \dfrac{2T}{h}$

excess pressure inside a drop

$\rightarrow h/2$

$h/2$

$\Delta P = T\left(\dfrac{1}{r_1} + \dfrac{1}{\infty}\right)$

$= \dfrac{T}{h/2} = \dfrac{2T}{h}$

$= T\left(\dfrac{1}{r_1} + \dfrac{1}{r_2}\right)$

Spherical. $r_1 = r_2 = R$

$\Delta P = \dfrac{2T}{R}$

bubble $\dfrac{4T}{R}$

$= \sqrt{\dfrac{2 \times 7 \times 10^{-2}}{10^3 \times 10}} = \sqrt{14} \times 10^{-3}$ m

$= \sqrt{14}$ mm

3.

$3 \times 3 \;|\; 14.0000$

9

6 | 500

3.74