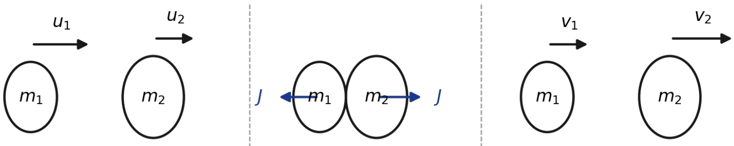
Head-on Collision — Formula Sheet

Class XI Physics · Centre of Mass & Collisions · valid for any coefficient of restitution $0 \le e \le 1$



before ($u_1 > u_2$) during: equal & opposite impulses after ($v_2 \ge v_1$)

Convention. Velocities are components along the line of impact, positive to the right. m_1 is the body *behind*, so $u_1 > u_2$ and the bodies approach. During the brief contact each body receives an impulse of the same magnitude J: backward on m_1, forward on m_2.

Impulse: $\mathbf{J = \int F\,dt = F_{av}\,\Delta t}$, Δt = duration of contact. Approach speed $\mathbf{v_{app} = u_1 - u_2}$; separation speed $\mathbf{v_{sep} = v_2 - v_1}$.

The six results

①	$v_{sep} = e\,v_{app} \Rightarrow \mathbf{v_2 - v_1 = e\,(u_1 - u_2)}$	Newton's law of restitution. Defines e; e = 1 perfectly elastic, e = 0 perfectly inelastic (bodies move together).
②	$J = \mu\,(v_{app} + v_{sep}) = \boldsymbol{\mu\,v_{app}\,(1 + e)}$	Impulse on either body. Follows from ①, ④, ⑤: subtracting ④ from ⑤ gives $v_2 - v_1 = -v_{app} + J/\mu$.
③	$\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ or $\dfrac{1}{\mu} = \dfrac{1}{m_1} + \dfrac{1}{m_2}$	Reduced mass. Always smaller than the lighter body; equals m/2 for equal masses m; ≈ lighter mass when the other is very heavy.
④	$p_{1f} = p_{1i} - J \Rightarrow \mathbf{v_1 = u_1 - J/m_1}$	Impulse–momentum for m_1 (impulse opposes its motion).
⑤	$p_{2f} = p_{2i} + J \Rightarrow \mathbf{v_2 = u_2 + J/m_2}$	Impulse–momentum for m_2. Adding $m_1\times$④ and $m_2\times$⑤ recovers momentum conservation: J cancels.
⑥	$\Delta K = \frac{1}{2}\,\mu\,(v_{app}{}^2 - v_{sep}{}^2) = \boldsymbol{\frac{1}{2}\,\mu\,v_{app}{}^2\,(1 - e^2)}$	Loss of kinetic energy. Only the KE of relative motion, $\frac{1}{2}\,\mu\,v_{rel}{}^2$, can change; the KE of the centre of mass is untouched.

Handy corollaries (all follow from ① – ⑥)

7	$v_1 = \dfrac{(m_1 - e\,m_2)\,u_1 + (1 + e)\,m_2\,u_2}{m_1 + m_2}$ $v_2 = \dfrac{(m_2 - e\,m_1)\,u_2 + (1 + e)\,m_1\,u_1}{m_1 + m_2}$	Closed form: put ② into ④ and ⑤. Use when a direct answer is asked; use ④/⑤ when impulse or force is asked.
8	$v_{cm} = \dfrac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$ unchanged, for every e	No external impulse along the line of impact. For e = 0 both bodies end up moving at v_{cm}.
9	$\Delta K = \frac{1}{2}\,J\,v_{app}\,(1 - e)$	Same as ⑥, written with the impulse — handy when J is already known.
10	KE fraction given to m_2 (at rest, e = 1) $= \dfrac{4\,m_1\,m_2}{(m_1 + m_2)^2}$	Maximum (100 %) for equal masses — the basis of choosing light nuclei as neutron moderators.

Special cases to recognise on sight

- **e = 1 (elastic):** $\Delta K = 0$; $v_{sep} = v_{app}$; $J = 2\mu\,v_{app}$. Equal masses ⇒ velocities are *exchanged*. Heavy on light at rest ⇒ light body leaves at ≈ 2u; light on heavy at rest ⇒ rebounds at ≈ u.
- **e = 0 (perfectly inelastic):** $v_1 = v_2 = v_{cm}$; $J = \mu\,v_{app}$; $\Delta K = \frac{1}{2}\,\mu\,v_{app}{}^2$ — the *maximum* possible loss for the given system.
- **m_1 stops dead** after hitting m_2 at rest only if $e = m_1/m_2$ (so it needs $m_1 \le m_2$).
- **Ball on fixed floor** ($m_2 \to \infty$): rebound speed = e × impact speed; rebound height = e^2 × drop height; total distance travelled before stopping $= h\,(1 + e^2)/(1 - e^2)$.