Answer Key & Model Answers

Practice Examination — Class XII Physics (Book I)

Session 2026–27 · Maximum Marks: 70

Each answer below is written the way a student should present it in the examination — the principle stated first, then the working set out step by step, with the diagram wherever one earns marks. Marks in the margin show how the examiner would split the credit.

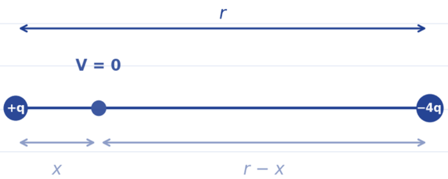
SECTION A — Multiple Choice and Assertion-Reason (1 mark each)

1. (b) $r/5$

Let the required point lie at a distance x from $+q$, between the two charges. Potential is a scalar, so the two contributions simply add:

$$V = kq/x + k(-4q)/(r - x) = 0 \implies q/x = 4q/(r - x)$$

$$r - x = 4x \implies 5x = r \implies x = r/5$$

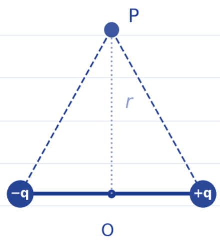


Teaching point: for two unlike charges there are in fact two zero-potential points — one between them ($x = r/5$) and one outside, beyond $+q$, at $r/3$ from it. The question asks for the point between the charges, so $r/5$ is the answer. Students should always check whether the question restricts the region.

2. (a) zero

Potential is a scalar, so the contributions of the two charges simply add algebraically. Every point on the equatorial line is equidistant from $+q$ and from $-q$.

P lies on the equatorial line of the dipole



Both charges are equidistant from P

Calling that common distance d, the potential at P is

$$V = (1/4\pi\varepsilon_0)(q/d) + (1/4\pi\varepsilon_0)(-q/d) = 0$$

The two contributions are equal in magnitude and opposite in sign, so they cancel exactly, and $V = 0$ at every point of the equatorial plane.

Contrast this with the electric FIELD, which is not zero on the equatorial line — it is $p/(4\pi\varepsilon_0 r^3)$, directed antiparallel to p. Potential is a scalar and cancels; field is a vector and the two contributions add to a non-zero resultant. Students routinely confuse the two.

3. (a) $v_d \propto E$

From $v_d = eE\tau/m$, with the relaxation time τ, the electron mass m and the charge e all constant for a given conductor at a fixed temperature, the drift velocity is directly proportional to E. This linear relation is exactly what makes Ohm's law hold in metals.

4. (d) 4R

Stretching does not change the volume of the wire, so $A'L' = AL$. With $L' = 2L$ we get $A' = A/2$.

$$R' = \rho L'/A' = \rho(2L)/(A/2) = 4(\rho L/A) = 4R$$

General result: on stretching to n times the original length, $R' = n^2 R$.

5. (b) 4950 Ω

For a voltmeter, a high resistance R is placed in series so that the full-scale current I_g flows when V is applied:

$$V = I_g (R + G) \implies R = V/I_g - G$$

$$R = 10/(2 \times 10^{-3}) - 50 = 5000 - 50 = 4950\ \Omega$$

6. (b) $E = -(v \times B)$

The net force is the Lorentz force $F = q(E + v \times B)$. For this to vanish for a non-zero charge, $E + v \times B = 0$, i.e. $E = -(v \times B)$. This is the condition used in a velocity selector.

7. (b) $\chi \propto 1/T$

Curie's law for a paramagnetic material states $\chi = C/T$, where C is the Curie constant. Physically, raising the temperature increases the random thermal agitation of the atomic dipoles, which opposes their alignment with the applied field, so the magnetisation and hence χ falls.

8. (b) a time-varying electric field

Maxwell showed that a changing electric flux produces a magnetic field just as a conduction current does. The associated displacement current is $I_d = \varepsilon_0 (d\Phi_E/dt)$, which is non-zero only where the electric field is changing with time.

9. (b) 24 V

The induced emf is the rate of change of flux:

$$\varepsilon = -d\Phi/dt = -(15t^2 - 20t + 4)$$

At $t = 2$ s:

$$|\varepsilon| = |15(4) - 20(2) + 4| = |60 - 40 + 4| = 24\ \text{V}$$

10. (c) $\frac{1}{2}LI^2$

Work must be done against the back emf while the current is being established. That work, $U = \frac{1}{2}LI^2$, is stored in the magnetic field of the coil — the magnetic analogue of $\frac{1}{2}CV^2$ for a capacitor.

11. (a) 0 rad

At resonance $X_L = X_C$, so the reactive parts cancel and $Z = R$. The circuit behaves as if purely resistive, and the current is exactly in phase with the applied voltage.

12. (b) 200 V

The rms value of an alternating voltage is $V_{rms} = V_0/\sqrt{2}$.
Dividing by $\sqrt{2}$ by hand is awkward, so turn the division into a multiplication. Since

$$1/\sqrt{2} = 0.707 \qquad (\text{and } \sqrt{2} = 1.414)$$

we may write

$$V_{rms} = V_0 \times (1/\sqrt{2}) = V_0 \times 0.707$$
$$V_{rms} = 282 \times 0.707$$

Splitting the multiplication into two easy parts:

$$282 \times 0.7 = 197.4 \qquad 282 \times 0.007 = 1.974$$
$$V_{rms} = 197.4 + 1.974 = 199.4 \approx 200\ \text{V}$$

Learn both constants by heart: $\sqrt{2} = 1.414$ and $1/\sqrt{2} = 0.707$. Whenever a formula tells you to divide by $\sqrt{2}$, multiply by 0.707 instead — in an examination without a calculator a short multiplication is far quicker, and much safer, than a long division. The same trick turns $I_0 = I_{rms}\sqrt{2}$ into $I_0 = I_{rms} \times 1.414$.

13. (a) Both A and R are true and R is the correct explanation of A

The tangent to a field line at a point gives the direction of E there. If two lines crossed, two different tangents — hence two directions of E — would exist at the same point, which is impossible for a well-defined vector field. R is exactly the reason for A.

14. (a) Both A and R are true and R is the correct explanation of A

Both statements are correct, and R explains A precisely. The drift velocity really is tiny — of the order of 10^{-4} m/s — so an individual electron would take hours to travel from the switch to the bulb. But the bulb does not wait for electrons to arrive from the switch.
On closing the switch, the electric field is set up throughout the conductor at a speed approaching c. Since the conductor already contains free electrons everywhere along its length, including inside the bulb filament, all of them start drifting almost at the same instant. The current is therefore established throughout the circuit almost immediately.

A useful analogy for the classroom: a long pipe already full of water. Push water in at one end and water leaves the far end at once — not because a molecule travelled the length of the pipe, but because the disturbance travelled through the medium already present.

15. (c) A is true but R is false

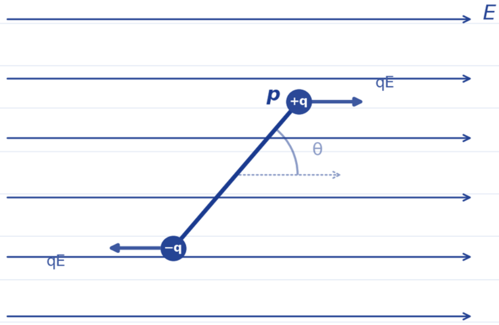
A is correct: eddy currents do damp the swinging plate — this is electromagnetic damping. R is wrong: by Lenz's law eddy currents always flow so as to OPPOSE the change of flux that produces them, never to increase it. It is precisely this opposition that produces the retarding force.

16. (d) A is false but R is true

A is false: a transformer does not create energy. In an ideal transformer the output power equals the input power, $V_p I_p = V_s I_s$. R is a true statement — stepping the voltage up by k steps the current down by k — and it is in fact the reason A is false.

SECTION B — Very Short Answer (2 marks each)

17. Potential energy of a dipole in a uniform field



The two equal and opposite forces qE on the charges form a couple whose torque is $\tau = pE \sin\theta$.

Torque expression … ½ mark

To turn the dipole through a further small angle $d\theta$ against this torque, the work done is

$$dW = \tau\, d\theta = pE \sin\theta\, d\theta$$

Element of work … ½ mark

Taking the zero of potential energy at $\theta = 90°$ (the position where the dipole is perpendicular to the field), the total work in turning it from 90° to θ is

$$W = \int pE \sin\theta\, d\theta \text{ from } 90° \text{ to } \theta = pE[-\cos\theta] \text{ from } 90° \text{ to } \theta = -pE\cos\theta$$

Integration … ½ mark

This work is stored as potential energy, so

$$U = -pE\cos\theta = -\mathbf{p}\cdot\mathbf{E}$$

Final result … ½ mark

Check the physics: U is minimum (−pE) at θ = 0°, the stable position with p along E; it is maximum (+pE) at θ = 180°, which is unstable.

18. Kirchhoff's rules

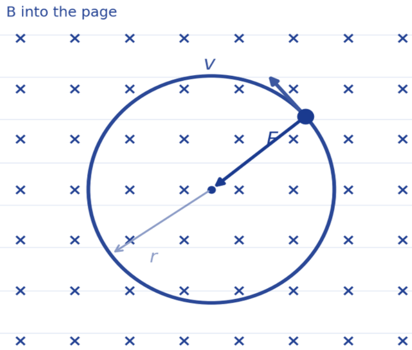
Junction rule: at any junction in a circuit, the algebraic sum of the currents is zero, $\Sigma I = 0$ — that is, the current flowing in equals the current flowing out. This follows from the conservation of electric charge, since charge cannot pile up at a junction.

Statement + principle … 1 mark

Loop rule: around any closed loop of a circuit, the algebraic sum of the changes in potential is zero, $\Sigma\Delta V = 0$. This follows from the conservation of energy, since a unit charge taken once round a loop and returned to its starting point must have no net change in potential energy.

Statement + principle … 1 mark

19. Speed of the electron (v ≈ 3.5 × 10⁷ m/s)



Magnetic force supplies the centripetal force

The magnetic force qvB acts at right angles to the velocity and so supplies exactly the centripetal force needed for circular motion:

$$qvB = mv^2/r \implies v = qBr/m$$

Formula ... 1 mark

Substituting $q = 1.6 \times 10^{-19}$ C, $B = 0.01$ T, $r = 0.02$ m and $m = 9.1 \times 10^{-31}$ kg:

$$v = (1.6 \times 10^{-19} \times 0.01 \times 0.02) / (9.1 \times 10^{-31}) = (3.2 \times 10^{-23})/(9.1 \times 10^{-31})$$

Handle the powers of ten first: $10^{-23} \div 10^{-31} = 10^8$. That leaves only $3.2 \div 9.1$ to evaluate.

Dividing by 9.1 is awkward, so divide by 9 and correct for the extra 0.1 using the binomial approximation:

$$3.2/9.1 = (3.2/9) \times 1/(1 + 0.0111) \approx 0.3556 \times (1 - 0.0111)$$
$$= 0.3556 - 0.0039 = 0.3517$$

A neater route is to factorise the divisor. Write the fraction as 32/91 and notice that $91 = 7 \times 13$. Dividing top and bottom by 13 leaves a division by 7, which is easy:

$$32/91 = (32 \div 13)/(91 \div 13) = 2.4615/7 = 0.3516$$

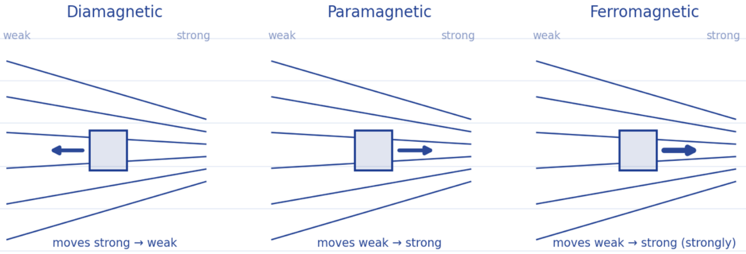
The two methods agree to three figures, which is a good check on both. Hence

$$v = 0.352 \times 10^8 \approx 3.5 \times 10^7 \text{ m/s}$$

Substitution and answer ... 1 mark

Always sanity-check such an answer against $c = 3 \times 10^8$ m/s. Here v is about a tenth of the speed of light, so the non-relativistic formula is acceptable. If a calculation of this kind ever gives a speed above c, the data must be wrong.

20. Diamagnetic, paramagnetic and ferromagnetic materials



(i) Susceptibility: diamagnetic — small and negative ($-1 \le \chi < 0$); paramagnetic — small and positive (χ slightly greater than 0); ferromagnetic — very large and positive ($\chi \gg 1$).

Susceptibility ... 1 mark

(ii) In a non-uniform field: a diamagnetic sample is pushed from the stronger towards the weaker part of the field (feeble repulsion); a paramagnetic sample moves from the weaker towards the stronger part (feeble attraction); a ferromagnetic sample is drawn strongly towards the stronger part of the field.

Behaviour in non-uniform field ... 1 mark

Examples worth quoting: bismuth, copper, water (diamagnetic); aluminium, sodium, oxygen (paramagnetic); iron, cobalt, nickel (ferromagnetic).

21. Mutual inductance

Definition: the mutual inductance of a pair of coils is the property by which a changing current in one coil induces an emf in the other. If a current I_1 in the first coil links a flux Φ_2 with the second, then $\Phi_2 = MI_1$, and the emf induced in the second coil is

$$\varepsilon_2 = -M\,(dI_1/dt)$$

So M may be defined as the flux linked with the second coil per unit current in the first, or as the emf induced in the second coil per unit rate of change of current in the first.

Definition … ½ mark

SI unit: the henry (H). Two coils have a mutual inductance of one henry if a current changing at 1 A s^{-1} in one of them induces an emf of 1 V in the other.

Unit … ½ mark

For two long coaxial solenoids of length l, area A, with N_1 and N_2 turns, $M = \mu_0\mu_r N_1N_2A/l$. Hence M depends on: (i) the number of turns on each solenoid; (ii) their common cross-sectional area A and length l, that is on their geometry; and (iii) the relative permeability μ_r of the core material. Any two of these are required.

Two factors … 1 mark

Mutual inductance also depends on how the coils are placed relative to each other — the coupling. Coaxial solenoids, one wound over the other, give the largest M because nearly all the flux of one links the other.

OR

Given: the current falls from 5.0 A to 0 in 0.1 s, and the induced emf is 200 V.

$$dI/dt = (5.0 - 0)/0.1 = 50\ A\ s^{-1}$$

Rate of change of current … 1 mark

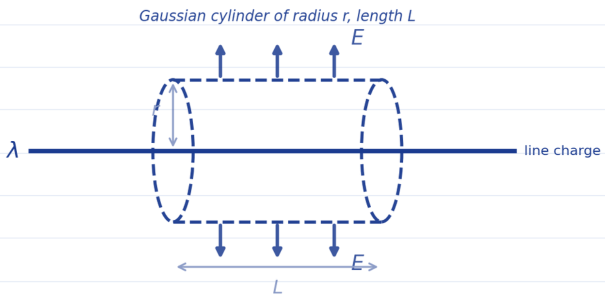
From $|\varepsilon| = L\ |dI/dt|$:

$$L = |\varepsilon|\ /\ |dI/dt| = 200/50 = 4\ H$$

Self-inductance … 1 mark

SECTION C — Short Answer (3 marks each)

22. Field due to an infinitely long charged wire



By symmetry the field must point radially outward from the wire (for $\lambda > 0$) and have the same magnitude at every point at the same distance r. Choose as Gaussian surface a cylinder of radius r and length L, coaxial with the wire.

Diagram and choice of Gaussian surface … 1 mark

Flux through the two flat ends is zero, because there E is parallel to the surface (the angle between E and the area vector is 90°). The whole flux passes through the curved surface, where E is everywhere perpendicular to the surface:

$$\Phi = E \times (2\pi rL)$$

Flux calculation … 1 mark

The charge enclosed by the cylinder is $q = \lambda L$. Gauss's theorem, $\Phi = q/\varepsilon_0$, then gives

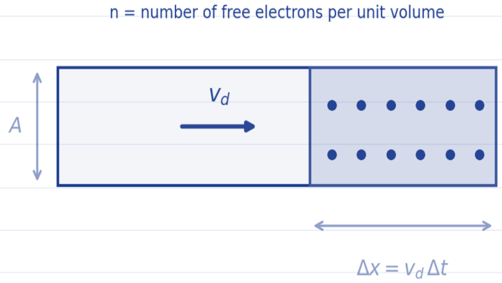
$$E\ (2\pi rL) = \lambda L/\varepsilon_0$$

$$E = \lambda / (2\pi\varepsilon_0 r)$$

Final expression … 1 mark

Note the distance dependence: $E \propto 1/r$ for a line charge, compared with $1/r^2$ for a point charge and a constant field for an infinite plane sheet. The geometry of the source determines the falloff.

23. Drift velocity and current



Definition: drift velocity is the small average velocity with which the free electrons in a conductor move opposite to the applied electric field, superposed on their much faster random thermal motion.

Definition … 1 mark

Let n be the number of free electrons per unit volume, A the cross-sectional area and v_d the drift velocity. In a time Δt every electron advances a distance

$$\Delta x = v_d \, \Delta t$$

So all the electrons in a segment of volume $A\,\Delta x = A\,v_d\,\Delta t$ cross the far face in that time. The number of such electrons is $n\,A\,v_d\,\Delta t$, and the charge crossing is

$$\Delta q = n\,e\,A\,v_d\,\Delta t$$

Charge crossing a section … 1 mark

Therefore the current is

$$I = \Delta q/\Delta t = n\,e\,A\,v_d$$

Final relation … 1 mark

24. Wheatstone bridge

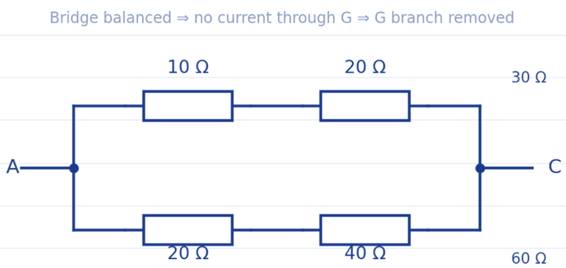
(a) The galvanometer shows no deflection, so the bridge is balanced and B and E are at the same potential. The balance condition is

$$10/X = 20/40$$

$$X = (10 \times 40)/20 = 20\ \Omega$$

Balance condition and X … 2 marks

(b) At balance no current passes through the galvanometer, so that branch may be removed. The upper arm and the lower arm are then simply two resistances in parallel across the battery:



$$\text{Upper arm} = 10 + 20 = 30\ \Omega, \quad \text{Lower arm} = 20 + 40 = 60\ \Omega$$
$$R_{eq} = (30 \times 60)/(30 + 60) = 1800/90 = 20\ \Omega$$
$$I = V/R_{eq} = 6/20 = 0.3\ A$$

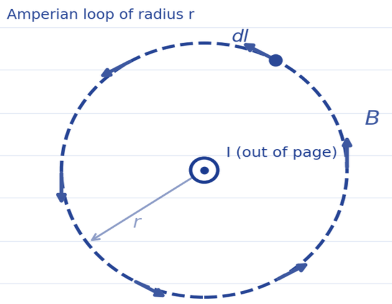
Equivalent resistance and current ... 1 mark

25. Ampère's circuital law and the field of a long straight wire

Statement: the line integral of the magnetic field around any closed loop equals μ_0 times the total current threading that loop.

$$\oint B \cdot dl = \mu_0\, I_{enclosed}$$

Statement ... 1 mark



B is tangential to the loop and constant in magnitude on it

Consider a long straight conductor carrying a steady current I. From the symmetry of the arrangement, the field lines must be circles concentric with the wire, and B must have the same magnitude at every point of any one such circle.

Choose as the Amperian loop a circle of radius r, centred on the wire and lying in the plane perpendicular to it. At every point of this loop B is tangential, and so parallel to the element dl. Hence $B \cdot dl = B\,dl$, and B may be taken outside the integral:

$$\oint B \cdot dl = B \oint dl = B\,(2\pi r)$$

Choice of loop and evaluation ... 1 mark

The whole current I threads the loop, so applying Ampère's law:

$$B\,(2\pi r) = \mu_0 I$$
$$B = \mu_0 I\ /\ (2\pi r)$$

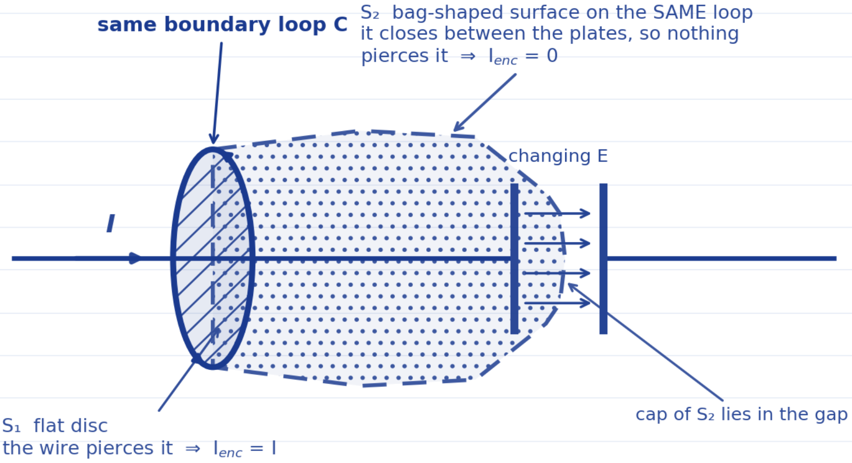
The direction is given by the right-hand thumb rule.

Final expression ... 1 mark

Compare this with Q22. In both cases the method is the same: exploit the symmetry to argue that the field is constant on a well-chosen closed path or surface, then let the integral collapse to a simple product. Notice too that both fields fall off as 1/r for an infinitely long source — one electric, one magnetic.

26. Displacement current and the Ampère–Maxwell law

(a) Ampère's law in its original form is $\oint B \cdot dl = \mu_0 I$, where I is the current threading the loop. The trouble is that the law says nothing about WHICH surface bounded by the loop we should use — and for a capacitor being charged, two equally valid choices give two different answers.



Same loop, two surfaces, two different answers — so the law is incomplete.

Take the circular loop C drawn round the wire, and consider two surfaces that share this same rim:

· S_1, the flat disc filling the loop. The wire passes straight through it, so the enclosed current is I and the law gives $\oint B\cdot dl = \mu_0 I$.

· S_2, a bag-shaped surface stretched from the same rim, swelling out past the left plate and closing off in the gap between the plates. The wire enters through the mouth of the bag and stops on the plate inside it, so no conduction current crosses S_2 anywhere. The law now gives $\oint B\cdot dl = 0$.

Both surfaces are bounded by the identical loop, so the left-hand side $\oint B\cdot dl$ is the same in each case — yet the right-hand side differs. The law as it stands is therefore incomplete.

The point to grasp is that the boundary loop is fixed; only the surface spanning it changes. A law about a loop cannot depend on which soap film you happen to hang on that loop. Maxwell's repair is to add a second term that is non-zero exactly where the conduction current stops — in the gap.

Inconsistency explained ... 1 mark

(b) Maxwell resolved this by noting that although no charge crosses the gap, the electric field between the plates is changing. He defined the displacement current

$$I_d = \varepsilon_0\, d\Phi_E/dt$$

where Φ_E is the electric flux between the plates. Between the plates, I_d is exactly equal to the conduction current in the wire, so the total current is continuous everywhere.

Definition and expression ... 1 mark

(c) The modified law, called the Ampère–Maxwell law, is

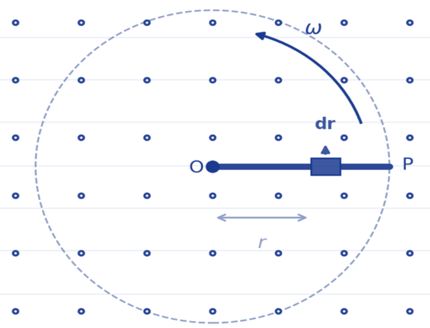
$$\oint B\cdot dl = \mu_0\,(I_c + I_d) = \mu_0\, I_c + \mu_0\varepsilon_0\, d\Phi_E/dt$$

Modified law ... 1 mark

This removes the inconsistency. On S_1 the enclosed current is the conduction current I; on S_2 there is no conduction current, but the changing electric flux through its cap gives a displacement current of exactly the same size, $I_d = I$. Both surfaces now give $\oint B\cdot dl = \mu_0 I$, as they must.

This modification is what completes Maxwell's equations and predicts electromagnetic waves: a changing electric field creates a magnetic field, and by Faraday's law a changing magnetic field creates an electric field, so the two can sustain each other and propagate.

27. Emf induced in a rotating rod



Consider a small element of the rod of length dr, at a distance r from the axis O. Because the rod rotates rigidly, the linear speed of this element is

$$v = \omega r$$

Speed of the element ... 1 mark

This element is a short conductor of length dr moving with speed v perpendicular to B, so the motional emf across it is

$$d\varepsilon = B\, v\, dr = B\, (\omega r)\, dr$$

Emf of the element ... 1 mark

Every element of the rod contributes an emf in the same sense, so the total emf between the ends O and P is obtained by integrating from r = 0 to r = L:

$$\varepsilon = \int B\omega r\, dr \text{ from 0 to L } = B\omega\, [r^2/2] \text{ from 0 to L}$$

$$\varepsilon = \tfrac{1}{2} B\, \omega\, L^2$$

Integration and result ... 1 mark

Equivalent view: in one revolution the rod sweeps an area πL^2, so the flux swept per unit time is $B(\pi L^2)(\omega/2\pi) = \tfrac{1}{2}B\omega L^2$, which is the same result.

28. Series LCR circuit

(a) Inductive and capacitive reactances:

$$X_L = \omega L = 1000 \times 0.1 = 100\ \Omega$$

$$X_C = 1/(\omega C) = 1/(1000 \times 5 \times 10^{-6}) = 1/(5 \times 10^{-3}) = 200\ \Omega$$

Both reactances ... 1 mark

(b) Since $X_C > X_L$, the circuit is capacitive. The impedance is

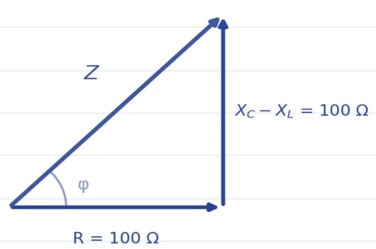
$$Z = \sqrt{[R^2 + (X_C - X_L)^2]} = \sqrt{[(100)^2 + (200 - 100)^2]}$$

$$Z = \sqrt{(10000 + 10000)} = \sqrt{20000}$$

Do not try to extract this root directly. Pull out the perfect square first:

$$\sqrt{20000} = \sqrt{(10000 \times 2)} = 100\sqrt{2} = 100 \times 1.414 \approx 141\ \Omega$$

Impedance ... 1 mark



(c) The rms current and the phase angle:

$$I_{rms} = V_{rms}/Z = 141/141 = 1.0 \text{ A}$$

$$\tan\varphi = (X_C - X_L)/R = 100/100 = 1 \implies \varphi = 45°$$

Because X_C exceeds X_L, the circuit is capacitive and the current leads the applied voltage by 45°.

Current, phase angle and which leads ... 1 mark

OR

Q-factor: the quality factor of a series resonant circuit measures the sharpness of resonance. It is defined as the ratio of the voltage developed across the inductor (or the capacitor) at resonance to the voltage across the resistor, equivalently

$$Q = \omega_0 L/R$$

Definition ... 1 mark

At resonance $\omega_0 = 1/\sqrt{LC}$. Substituting,

$$Q = \omega_0 L/R = (1/\sqrt{LC})(L/R) = (1/R)\sqrt{L/C}$$

Derivation ... 1 mark

Physical significance: a large Q means a tall, narrow resonance peak, so the circuit responds strongly to a very narrow band of frequencies around ω_0 and rejects the rest. This selectivity is what allows a radio tuning circuit to pick out one station and reject the neighbouring ones.

Significance ... 1 mark

SECTION D — Case-Based Questions (4 marks each)

29. Dielectrics and capacitors

(i) The polarisation vector P is the electric dipole moment induced per unit volume of the dielectric. For a linear isotropic dielectric it is proportional to the applied field:

$$P = \chi_e \varepsilon_0 E$$

where χ_e is the electric susceptibility of the medium, related to the dielectric constant by $K = 1 + \chi_e$.

1 mark

(ii) The battery stays connected, so the potential difference V is held constant while the capacitance rises to $C' = KC = 5C$. The stored energy is

$$U' = \tfrac{1}{2}C'V^2 = \tfrac{1}{2}(5C)V^2 = 5U$$

The energy stored becomes five times its original value; the extra energy is supplied by the battery, which pushes more charge onto the plates.

1 mark

(iii) Here the battery is disconnected first, so the charge Q is now the constant quantity, not V.

$$U_i = Q^2/(2C) \qquad U_f = Q^2/(2KC) = U_i/K$$

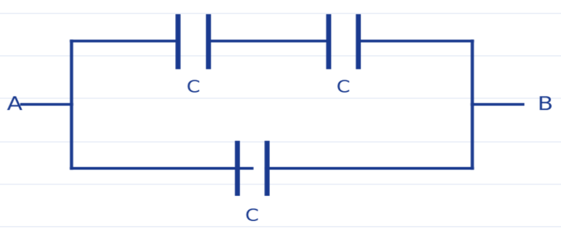
$$U_i : U_f = K : 1$$

The stored energy falls by the factor K. The energy is used up in polarising the dielectric — in fact the slab is pulled into the gap, and the field does that work.

2 marks

Compare (ii) and (iii) carefully. Whether the battery stays connected decides which quantity is held fixed — V or Q — and the two cases give opposite answers. Deciding this first is the whole skill in such problems.

OR (iii) Effective capacitance of the network



The two capacitors in the upper arm are in series:

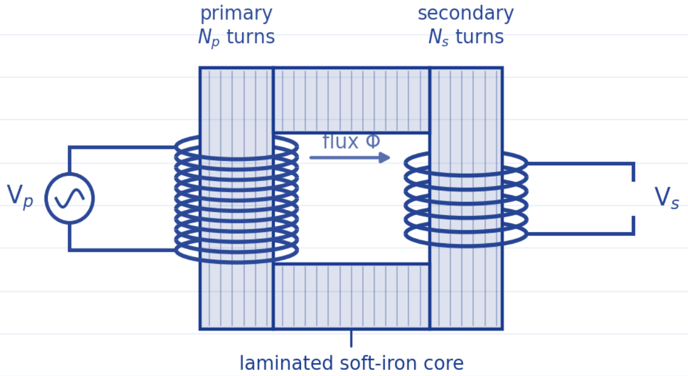
$$1/C_{series} = 1/C + 1/C = 2/C \implies C_{series} = C/2$$

This combination is in parallel with the third capacitor in the lower arm, so

$$C_{AB} = C/2 + C = 3C/2 = 1.5\ C$$

2 marks

30. Transformers in the power grid



Each winding is wound right round its own limb; the core carries the flux from one to the other.

(i) A transformer works on the principle of mutual induction: an alternating current in the primary coil produces a continually changing magnetic flux, which is carried by the soft-iron core and links the secondary coil, inducing an alternating emf in it.

1 mark

(ii) The core is laminated — built from thin sheets insulated from one another — to break up the paths available to eddy currents induced in the core. Thin laminations mean the induced eddy currents are confined to small loops of high resistance, so the I^2R heating loss in the core is greatly reduced.

1 mark

(iii) For an ideal transformer, no power is lost, so the input power equals the output power:

$$V_p I_p = V_s I_s = 4400 \text{ W}$$

$$I_p = 4400/2200 = 2 \text{ A}$$

The turns ratio follows from $V_s/V_p = N_s/N_p$:

$$N_s/N_p = 220/2200 = 1/10$$

So the primary current is 2 A and the turns ratio is 1 : 10 — correctly a step-down transformer, having fewer turns in the secondary.

2 marks

OR (iii) Two sources of loss and their remedies

Eddy current loss: currents induced in the body of the core dissipate energy as heat. Minimised by using a laminated core of thin insulated sheets, which raises the resistance of the eddy-current paths.
Hysteresis loss: energy is spent in repeatedly magnetising and demagnetising the core through each AC cycle. Minimised by making the core of soft iron, a material with a narrow hysteresis loop and therefore a small loop area per cycle.
(Copper loss in the winding resistance — reduced by using thick copper wire of low resistance — and flux leakage — reduced by winding the coils one over the other on the same core — are also acceptable.)

2 marks (1 mark each)

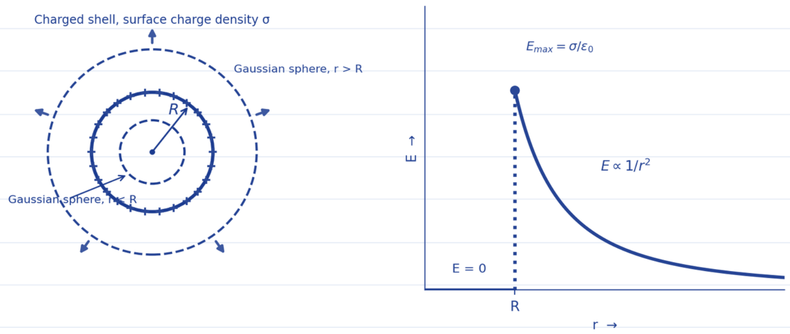
SECTION E — Long Answer (5 marks each)

31. Gauss's law and the charged spherical shell

(a) Gauss's law: the total electric flux through any closed surface equals $1/\varepsilon_0$ times the net charge enclosed by that surface.

$$\oint E \cdot dS = q_{enclosed} / \varepsilon_0$$

Statement ... 1 mark



Let the shell carry a total charge $q = \sigma(4\pi R^2)$, spread uniformly over its surface. By symmetry the field must be radial and of equal magnitude at all points of any concentric sphere.
(i) Point outside the shell ($r > R$). Take a concentric Gaussian sphere of radius r. The whole charge is enclosed:

$$E\,(4\pi r^2) = q/\varepsilon_0$$
$$E = q/(4\pi\varepsilon_0 r^2) = \sigma R^2/(\varepsilon_0 r^2)$$

The shell therefore behaves, for all external points, exactly as if its entire charge were concentrated at its centre.

Field outside ... 2 marks

(ii) Point inside the shell ($r < R$). Now the Gaussian sphere lies wholly within the cavity and encloses no charge at all, since all the charge resides on the surface:

$$E\,(4\pi r^2) = 0/\varepsilon_0 \quad \Longrightarrow \quad E = 0$$

The field vanishes everywhere inside the shell — the basis of electrostatic shielding.

Field inside ... 1 mark

(b) The graph (shown alongside) is zero from the centre out to $r = R$, jumps to its maximum value σ/ε_0 at the surface, and thereafter falls off as $1/r^2$.

Graph ... 1 mark

OR

(a) Capacitance of a parallel plate capacitor. Let the plates each have area A and carry charges $+Q$ and $-Q$, separated by a distance d with air between them. The surface charge density is $\sigma = Q/A$.

Treating the plates as infinite sheets, the fields of the two plates add in the region between them and cancel outside, giving a uniform field

$$E = \sigma/\varepsilon_0 = Q/(\varepsilon_0 A)$$

The potential difference between the plates is therefore

$$V = Ed = Qd/(\varepsilon_0 A)$$

Hence the capacitance is

$$C = Q/V = \varepsilon_0 A/d$$

Derivation ... 3 marks

(b) Three capacitors 4 μF, 6 μF and 12 μF in series across 24 V.

$$1/C_{eq} = 1/4 + 1/6 + 1/12 = (3 + 2 + 1)/12 = 6/12 = 1/2 \quad \Longrightarrow \quad C_{eq} = 2\ \mu F$$

(ii) In a series combination the same charge appears on every capacitor:

$$Q = C_{eq}\ V = 2\ \mu F \times 24\ V = 48\ \mu C \text{ on each capacitor}$$

(iii) Potential difference across the 4 μF capacitor:

$$V_4 = Q/C = 48\ \mu C\ /\ 4\ \mu F = 12\ V$$

Three parts ... 2 marks

A useful check: the three potential differences are 48/4 = 12 V, 48/6 = 8 V and 48/12 = 4 V, and these add to 24 V as they must. The smallest capacitor always takes the largest share of the voltage.

32. Galvanometer sensitivity and its conversion

(a) In a moving coil galvanometer the coil hangs in a radial magnetic field, so the deflecting torque NIAB is balanced by the restoring torque kφ of the suspension:

$$NIAB = k\varphi \quad \Longrightarrow \quad \varphi = (NAB/k)\ I$$

Current sensitivity is the deflection produced per unit current:

$$I_s = \varphi/I = NAB/k$$

Current sensitivity ... 1 mark

Voltage sensitivity is the deflection produced per unit potential difference applied across the galvanometer, whose resistance is R:

$$V_s = \varphi/V = \varphi/(IR) = NAB/(kR) = I_s\ /\ R$$

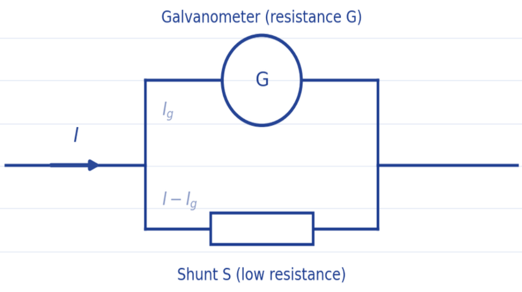
Voltage sensitivity ... 1 mark

Why more turns need not help: increasing N does raise the current sensitivity, since $I_s \propto N$. But adding turns also lengthens the wire of the coil, so its resistance R increases in very nearly the same proportion. Since $V_s = I_s/R$, the two effects largely cancel and the voltage sensitivity may hardly change — it can even fall.

Explanation ... 1 mark

The lesson is that the two sensitivities are not interchangeable. A galvanometer that is a sensitive current detector is not automatically a sensitive voltage detector.

(b)(i) Conversion into an ammeter. A low resistance S, the shunt, is connected in parallel with the galvanometer.



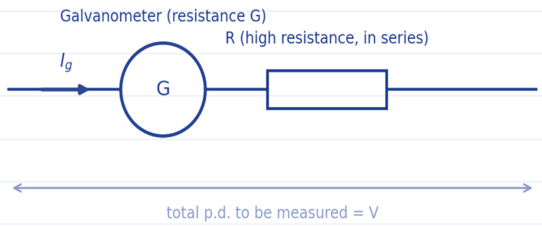
Only I_g passes through the galvanometer and the remaining $(I - I_g)$ through the shunt. Being in parallel, both carry the same potential difference:

$$I_g G = (I - I_g) S \implies S = I_g G / (I - I_g)$$

Since S is very small, the combined resistance is very low — as an ammeter's must be, so that connecting it in series does not appreciably alter the current it is measuring.

Ammeter ... 1 mark

(b)(ii) Conversion into a voltmeter. A high resistance R is connected in series with the galvanometer.



The full-scale current I_g must flow when the full range V is applied across the combination:

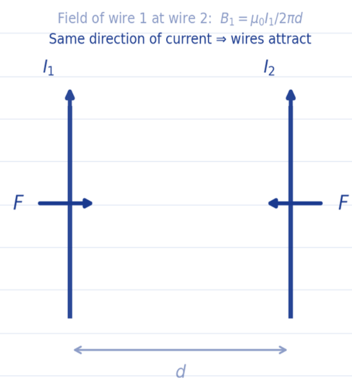
$$V = I_g (G + R) \implies R = V/I_g - G$$

Since R is very large, the combined resistance is high — as a voltmeter's must be, so that connecting it in parallel draws almost no current and does not disturb the potential difference being measured.

Voltmeter ... 1 mark

OR

(a) Force between two parallel current-carrying conductors.



Consider two long straight parallel conductors a distance d apart in air, carrying currents I_1 and I_2 in the same direction. The first conductor produces at the site of the second a magnetic field

$$B_1 = \mu_0 I_1/(2\pi d)$$

directed perpendicular to the second conductor. A length l of the second conductor, carrying current I_2 in this field, experiences a force

$$F = B_1 I_2 l = \mu_0 I_1 I_2 l/(2\pi d)$$

so that the force per unit length is

$$F/l = \mu_0 I_1 I_2 / (2\pi d)$$

Derivation ... 3 marks

Applying Fleming's left-hand rule, the force on the second conductor points towards the first. By Newton's third law the first is pulled equally towards the second. Hence: conductors carrying currents in the SAME direction attract one another, and those carrying currents in OPPOSITE directions repel.

(b) Definition of the ampere: put $I_1 = I_2 = 1$ A and $d = 1$ m. Then

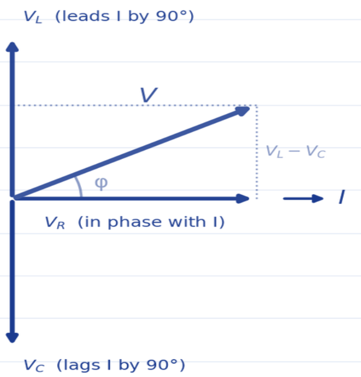
$$F/l = (4\pi \times 10^{-7} \times 1 \times 1)/(2\pi \times 1) = 2 \times 10^{-7} \text{ N/m}$$

One ampere is therefore that steady current which, when maintained in each of two infinitely long straight parallel conductors of negligible cross-section placed 1 m apart in vacuum, produces a force of 2×10^{-7} newton per metre of their length.

Definition … 2 marks

33. Series LCR circuit by phasors, and power in AC circuits

(a) In a series circuit the same current $I = I_0 \sin \omega t$ flows through every element, so the current is taken as the reference phasor.



Across the resistor, $V_R = I\,R$ is in phase with the current. Across the inductor, $V_L = I\,X_L$ leads the current by 90°. Across the capacitor, $V_C = I\,X_C$ lags the current by 90°.

Phasor diagram … 1 mark

V_L and V_C are opposite in direction, so they partly cancel and their resultant has magnitude $|V_L - V_C|$, perpendicular to V_R. The applied voltage is the vector sum:

$$V^2 = V_R^2 + (V_L - V_C)^2 = I^2R^2 + I^2(X_L - X_C)^2$$

$$V = I\,\sqrt{[R^2 + (X_L - X_C)^2]}$$

Resultant voltage … 1 mark

Hence the impedance is

$$Z = V/I = \sqrt{[R^2 + (X_L - X_C)^2]}, \quad \text{where } X_L = \omega L \text{ and } X_C = 1/\omega C$$

and the phase difference φ between the voltage and the current is given by

$$\tan\varphi = (X_L - X_C)/R$$

Impedance and phase angle … 1 mark

(b) Average power in a series LCR circuit. Let the applied voltage and the resulting current be

$$V = V_0 \sin\omega t \qquad \text{and} \qquad I = I_0 \sin(\omega t - \varphi)$$

The instantaneous power is $P = VI = V_0I_0 \sin\omega t \cdot \sin(\omega t - \varphi)$. Expanding the product,

$$P = (V_0I_0/2)\,[\,\cos\varphi - \cos(2\omega t - \varphi)\,]$$

Averaged over one complete cycle, the term $\cos(2\omega t - \varphi)$ oscillates symmetrically about zero and averages to zero, while $\cos\varphi$ is a constant. Hence

$$P_{av} = (V_0I_0/2)\cos\varphi = (V_0/\sqrt{2})(I_0/\sqrt{2})\cos\varphi$$

$$P_{av} = V_{rms}\,I_{rms}\cos\varphi$$

Derivation of average power … 2 marks

Power factor: the quantity $\cos\varphi$ is called the power factor of the circuit. It is the ratio of the actual power dissipated to the apparent power $V_{rms}\,I_{rms}$, and from the impedance triangle it equals R/Z.

Power factor … 1 mark

Wattless current: if the circuit contains only a pure inductor or only a pure capacitor, then $\varphi = 90°$ and $\cos\varphi = 0$, so $P_{av} = 0$. A current flows, and may be large, yet no power is dissipated over a cycle

— energy is drawn from the source during one quarter cycle and returned to it during the next. Such a current is called a wattless or idle current, and the component of the current that is at 90° to the voltage is its wattless component.

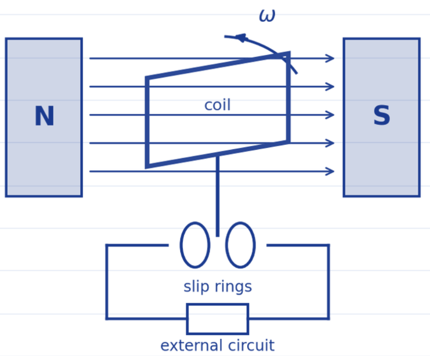
Wattless current ... 1 mark

Practical consequence: in industry a low power factor means a large current is drawn for the useful power delivered, which wastes energy as I^2R heating in the supply cables. Power factor correction — usually by adding capacitors — is done for exactly this reason.

OR

(a) AC generator. It converts mechanical energy of rotation into electrical energy, and works on the principle of electromagnetic induction: when the magnetic flux linked with a coil changes, an emf is induced in it.

Coil rotates in a uniform magnetic field



Construction: a rectangular armature coil of N turns and area A is mounted on an axle between the poles of a strong magnet, which provides a uniform field B. The ends of the coil are joined to two slip rings that rotate with it, and carbon brushes press against these rings to carry the current to the external circuit.

Construction and diagram ... 2 marks

Theory: let the coil be rotated with a uniform angular velocity ω. If at time t the normal to the coil makes an angle $\theta = \omega t$ with B, the flux linked with the coil is

$$\Phi = NBA \cos \omega t$$

By Faraday's law the induced emf is

$$\varepsilon = -d\Phi/dt = -d(NBA \cos \omega t)/dt = NBA\omega \sin \omega t$$

$$\varepsilon = \varepsilon_0 \sin \omega t, \quad \text{where } \varepsilon_0 = NBA\omega$$

The emf therefore varies sinusoidally, reversing its direction twice in each rotation — an alternating emf.

Derivation ... 1 mark

(b) Numerical. N = 100, A = 0.1 m^2, B = 0.05 T, and 3000 revolutions per minute means

$$f = 3000/60 = 50 \text{ rev/s}, \quad \omega = 2\pi f = 2\pi \times 50 = 100\pi \approx 314 \text{ rad/s}$$

$$\varepsilon_0 = NBA\omega = 100 \times 0.05 \times 0.1 \times 314$$

$$\varepsilon_0 = 0.5 \times 314 = 157 \text{ V}$$

Numerical ... 2 marks

APPENDIX A — Working without a calculator

You are not allowed a calculator in the examination, so the arithmetic has to be arranged to suit the hand. Five habits will carry you through almost every numerical in this paper.

1. Turn every division into a multiplication

Memorise the reciprocal and multiply by it — a multiplication splits into easy parts, a long division does not.

$$V_0/\sqrt{2} = V_0 \times 0.707 \qquad I_{rms}\sqrt{2} = I_{rms} \times 1.414$$

Then split: $282 \times 0.707 = 282 \times 0.7 + 282 \times 0.007 = 197.4 + 1.974 = 199.4$.

2. Deal with the powers of ten separately

Separate the mantissa from the exponent, settle the exponent by inspection, and only then do the small division. In Q19, $10^{-23} \div 10^{-31} = 10^{8}$ immediately, leaving just $3.2 \div 9.1$ to work out.

3. Use the binomial approximation

For any small x, $(1 + x)^n \approx 1 + nx$. This one line handles both awkward roots and awkward divisions.

Roots — bring out the nearest perfect square, then correct:

$$\sqrt{50} = \sqrt{(49 \times 50/49)} = 7\sqrt{(1 + 1/49)} \approx 7(1 + 1/98) = 7.071$$

Divisions — divide by the convenient nearby number, then correct:

$$3.2/9.1 = (3.2/9) \times (1 + 0.0111)^{-1} \approx 0.3556 \times (1 - 0.0111) = 0.3517$$

And always pull perfect squares out of a surd before evaluating: $\sqrt{20000} = 100\sqrt{2} = 141.4$.

4. Factorise the divisor and divide in stages

An ugly divisor is often the product of two friendly ones. Clear the decimals first, then look for a factor you can cancel.

$$3.2/9.1 = 32/91, \quad \text{and} \quad 91 = 7 \times 13$$

$$32/91 = (32 \div 13)/(91 \div 13) = 2.4615/7 = 0.3516$$

Unlike the binomial, this method is exact — nothing is approximated. Where both routes are open, working the division twice is the surest check.

Divisors worth recognising on sight: $91 = 7 \times 13$, $143 = 11 \times 13$, $119 = 7 \times 17$, $133 = 7 \times 19$, $121 = 11^2$, $169 = 13^2$. It is also worth knowing $1/7 = 0.142857$ recurring, $1/11 = 0.0909...$, and $1/13 = 0.0769...$

5. Use the friendly forms of the constants

$$1/(4\pi\varepsilon_0) = 9 \times 10^{9}\ \text{N m}^2\ \text{C}^{-2}$$

$$\varepsilon_0 = 1/(36\pi \times 10^{9})\ \text{F m}^{-1}, \quad \text{so} \quad 1/\varepsilon_0 = 36\pi \times 10^{9}$$

$$\mu_0 = 4\pi \times 10^{-7}\ \text{T m A}^{-1}, \quad \text{so} \quad \mu_0/4\pi = 10^{-7}$$

Writing ε_0 as $1/(36\pi \times 10^{9})$ is almost always better than 8.85×10^{-12}, because the π usually cancels against a π elsewhere in the formula and leaves a round number behind.

Constants worth knowing by heart

$$\sqrt{2} = 1.414 \qquad 1/\sqrt{2} = 0.707 \qquad \sqrt{3} = 1.732 \qquad 1/\sqrt{3} = 0.577$$

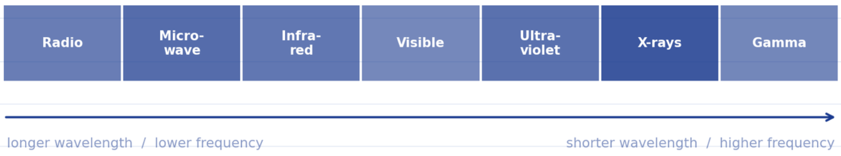
$$\sqrt{5} = 2.236 \qquad \sqrt{10} = 3.162 \qquad \pi = 3.142 \qquad 2\pi = 6.283 \qquad \pi^2 \approx 9.87$$

Finally, ask whether the answer is physically sensible. A speed above 3×10^{8} m/s, a negative resistance or an efficiency above 100% means an arithmetic slip — catching it costs seconds, losing the mark costs more.

APPENDIX B — The electromagnetic spectrum

Several questions in this paper draw on Chapter 8. The chart below is worth committing to memory in order, together with one use for each band.

The electromagnetic spectrum



Radio waves — radio and television broadcasting, and cellular communication. Microwaves — RADAR, satellite links and microwave ovens. Infrared — thermal imaging, remote controls, physiotherapy; also responsible for the greenhouse effect. Visible light — the narrow band the human eye detects. Ultraviolet — sterilisation of water and surgical instruments; largely absorbed by the ozone layer. X-rays — medical imaging and the study of crystal structure. Gamma rays — treatment of cancerous tumours, and the study of nuclear structure.

All of these travel in vacuum at the same speed $c = 3 \times 10^8$ m/s; they differ only in wavelength and frequency, related by $c = \nu\lambda$. In every electromagnetic wave, E and B oscillate in phase, perpendicular to each other and to the direction of propagation, with $E_0 = cB_0$.

— End of Answer Key —